# V Semester B.A./B.Sc. Examination, Nov./Dec. 2015
## (Semester Scheme) (2013-14 and Onwards) (NS)
## MATHEMATICS – V

Time : 3 Hours Max. Marks : 100

***Instruction :** Answer **all** questions.*

I. Answer **any fifteen** questions : **(15×2=30)**

1) In a vector space V(F) if $c\alpha = c\beta$ and $c \neq 0$ then show that $\alpha = \beta$ where $\alpha, \beta \in V$ and $c \in F$.

2) Define subspace of a vector space V(F).

3) Show that the set S = {(1, 2, 1), (–1, 1, 0), (5, –1, 2)} are linearly independent.



4) In a linear transformation $T : U \to V$ show that $T(-\alpha) = -T(\alpha)\ \forall\ \alpha \in U$.

5) Find the matrix of the linear transformation $T : V_2(R) \to V_3(R)$ defined by $T(x, y) = (x + y, x, 3x - y)$ relative to standard basis.

6) Define rank and nullity of linear transformation $T : U \to V$.

7) If $\vec{a}$ has a constant length then prove that $\vec{a}$ and $\frac{d\vec{a}}{dt}$ are perpendicular provided $\left|\frac{d\vec{a}}{dt}\right| \neq 0$.

8) If $\vec{r} = e^{nt}\vec{a} + e^{-nt}\vec{b}$ where $\vec{a}$ and $\vec{b}$ are constant vectors, then show that $\frac{d^2\vec{r}}{dt^2} - n^2\vec{r} = \vec{o}$.

9) Find the unit tangent vector $\hat{t}$ at t = 0 on the space curve $x = 3t$, $y = 3t^2$ and $z = 2t^3$.

10) Show that the necessary condition for a curve in space to be a straight line is that curvature K = 0 at all points.

11) The Cartesian coordinates of a point are (2, –2, 4). Find the corresponding cylindrical coordinates.

12) If $\phi = x^3 + y^3 + xz^2$ find $|\nabla\phi|$ at the point (1, –1, 2).

13) Show that $\overrightarrow{F} = (x + 3y)\hat{i} + (y - 3z)\hat{j} + (x - 2z)\hat{k}$ is solenoidal.

14) Find $\nabla^2\phi$, where $\phi = xy + yz + zx$.

15) Prove that curl (grad $\phi$) = 0.

16) If $\overrightarrow{F} = x^2y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$ find curl $\overrightarrow{F}$.

17) Define the complex Fourier transform and the inverse complex Fourier transform of a function f(x).



18) With the usual notations of Fourier transform prove that

$F[af(x) + bg(x)] = aF(f(x)) + bF(g(x))$

19) If a is any real constant and $F_s[f(x)] = \hat{f}_s(\alpha)$ then prove that $F_s[f(ax)] = \frac{1}{a}\hat{f}_s\left(\frac{\alpha}{a}\right)$.

20) If $\hat{f}(\alpha)$ is the Fourier transform of the function f(x). Then prove that

$F[f'(x)] = -i\alpha\,\hat{f}(\alpha)$.

II. Answer **any four** of the following : **(4×5=20)**

1) Prove that a non-empty subset W of a vector space V(F) is a subspace of V if and only if.

i) $\forall\ \alpha, \beta \in W \Rightarrow \alpha + \beta \in W$

ii) $a \in F, \alpha \in W \Rightarrow a\alpha \in W$.

2) Express the vector (3, 5, 2) as a linear combination of the vectors {(1, 1, 0), (2, 3, 0), (0, 0, 1)} of $V_3(R)$.

3) Find the dimension and basis of the subspace spanned by the vectors {(2, 4, 2), (1, –1, 0), (1, 2, 1), (0, 2, 1)} of $V_3(R)$.

4) Find the linear transformation $T: R^2 \to R^3$ such that $T(1, 1) = (0, 1, 2)$ and $T(-1, 1) = (2, 1, 0)$.

5) Verify Rank-nullity theorem for the transformation $T: R^3 \to R^3$ defined by $T(e_1) = e_1 - e_2$; $T(e_2) = 2e_1 + e_3$; $T(e_3) = e_1 + e_2 + e_3$.

6) Let $T: U \to V$ be a linear transformation, prove that if the vectors $\alpha_1, \alpha_2, ...., \alpha_n$ generates U then the vectors $T(\alpha_1), T(\alpha_2), ...., T(\alpha_n)$ generates R(T).

III. Answer **any four** of the following : (4×5=20)

1) For the space curve $x = a \cos t, y = a \sin t, z = bt$ show that $\kappa = \frac{a}{a^2+b^2}$ and $\tau = \frac{a}{a^2+b^2}$.

2) State and prove Serret-Frenet formulae.



3) Find the angle between the unit tangent vectors drawn to the curve $x = t^2$, $y = 2t, z = -t^3$ at the points $t = 1$ and $t = -1$.

4) Find the equation of the tangent plane and normal line to the surface $x^2 + y^2 + z^2 - 25 = 0$ at the point (4, 0, 3).

5) Show that the surface $5x^2 - 2yz = 9x$ is orthogonal to the surface $4x^2y + z^3 = 4$ at (1, –1, 2).

6) Express the vector $\vec{f} = 2y\hat{i} - z\hat{j} + 3x\hat{k}$ in terms of cylindrical co-ordinates.

IV. Answer **any three** of the following : (3×5=15)

1) Find the directional derivative of $\phi(x, y, z) = x^2 - 2y^2 + 4z^2$ at the point (1, 1, –1) in the direction of $2\hat{i} + \hat{j} - \hat{k}$.

2) Show that $\text{div}(\phi\vec{F}) = \phi(\text{div}\vec{F}) + \vec{F} \cdot (\text{grad}\,\phi)$.

3) Show that $\vec{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Find $\phi$ such that $\vec{F} = \nabla\phi$.

4) If $\vec{F} = 2x\hat{i} + 3y\hat{j} + 4z\hat{k}$ and $\phi = xy^2z^3$, find $\vec{F} \cdot \nabla\phi$ and $\nabla|\vec{F}|^2$.

5) Show that $\nabla \cdot \left[r\nabla\left(\frac{1}{r^3}\right)\right] = \frac{3}{r^4}$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

V. Answer **any three** of the following : (3×5=15)

1) Express $f(x) = \begin{cases} 1 & \text{for } |x| \le 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ as a Fourier integral.

2) Find the Fourier transform of $f(x) = e^{-|x|}$.

3) Find the Fourier cosine transform of the function $f(x) = \begin{cases} 1+x & \text{, for } 0 < x < 1 \\ 0 & \text{, for } x > 1 \end{cases}$

4) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$, $a > 0$.

5) Prove that :

i) $F_s[f'(x)] = -\alpha F_c[f'(x)]$

ii) $F_c[f'(x)] = -\sqrt{\frac{2}{\pi}}\, f(0) + \alpha\, F_s[f(x)]$.